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OPTIMAL ACTUATOR AND SENSOR PLACEMENT WITH REGARD TO COUPLED ELECTRO-MECHANICAL BEHAVIOUR OF SMART STRUCTURES

TAMARA NESTOROVIĆ* AND MIROSLAV TRAJKOV†

Ruhr-Universität Bochum, Mechanics of Adaptive Systems
Universitätsstraße 150, Geb. IA-01/128, D-44801 Bochum, Germany
*e-mail: tamara.nestorovic@rub.de
† email: miroslav.trajkov@rub.de

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Abstract. In this paper the problem of optimal actuator and sensor placement for active large flexible structures is considered. The proposed placement optimization method is based on balanced reduced models. It overcomes disadvantages arising from demanding numeric procedures related with high order structural models. Optimization procedure relies on H_2 and H_∞ norms, as well as on controllability and observability Gramians, related with structural eigenmodes of interest. The optimization procedure is documented by examples, which show a good agreement between the results obtained using different placement indices.

1 INTRODUCTION

The study and development of piezoelectric smart structures involves a very important investigation of optimal actuator and sensor placement. Especially for piezoelectric smart structures and systems, the placement once applied cannot be changed easily and it is often related with the need to build a new structure in order to perform another placement constellation for actuators and sensors. Development of appropriate and reliable optimization procedures, which can be applied prior to real structure or a prototype building, is therefore the task of a great significance. In this paper we have proposed a reliable method for determining appropriate actuator/sensor placement, based on structural models developed using the finite element (FE) approach. Model based approach represents an indispensable tool in the optimization procedure due to requirement for iterative problem solution.

Optimization problem was treated by several authors and investigated for different structures. An overview of the optimization criteria for optimal placement of piezoelectric sensors and actuators on a smart structure was given in a technical review by Gupta et al. [1]. In [2] based on the modal approach, optimal geometrical conditions were obtained for several cases of active beams with different boundary conditions. Optimization criterion for finding optimal actuator/sensor positions for piezoelectric beams in [3] is the performance of an optimal LQR controller. In [4] efficiency indices based on the mode shapes for a clamped piezoelectric beam were determined for typical eigenmodes.

Kumar and Narayanan [5] have applied the LQR controller based criteria to find optimal location of piezoelectric actuators/sensors for vibration control of plates and used genetic algorithm (GA) for solving a zero-one optimization problem. Peng et al. [6] involved maximizing of the controllability Gramian as the optimization criterion for optimal placement on a clamped plate using GA. Similar approach with modal controllability and observability Gramians and GA were also used in [7].

In this paper we present a general approach to optimal actuator and sensor placement applicable both for beam and plate structures, but also for other complex geometries of structures. The optimal placement procedure is based on the method for balanced model reduction, which assumes models with equally controllable and observable retained modes. The method has advantage over modal truncation and mathematical criteria for controllability and observability, since the retaining of the modes of interest is founded on their equal controllability and observability expressed in terms of appropriate Gramians. Further the paper deals with optimization criteria based on the H_2 and H_∞ norms, which are calculated for all possible candidate locations. In this way the fulfillment of the criteria is not limited to a narrow set of selected assumed favorable locations, but it relies on verification through all candidate positions by finding the placement indices with largest values.

2 MODELS AND OBJECTIVE FUNCTIONS FOR OPTIMAL PLACEMENT

The procedure for finding optimal placement of actuators and sensors relies on the state space models of smart structures, which are obtained through the finite element (FE) modeling procedure and model order reduction.

2.1. FE based state space models

Applying general FE modeling procedure the model of a smart structure can be represented as a set of equations of motion in matrix form (1) obtained by assembling all finite elements of the structure (more details on FE modeling of piezoelectric structures can be found in [8,9]).

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}_\mathrm{d}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F} \tag{1}$$

Vector q contains all degrees of freedom and it can be formed e.g. by node-wise arranging of degrees of freedom for all elements. For modeling of piezoelectric materials besides mechanical degrees of freedom, electric voltage or charge is included as additional degree of freedom to model electro-mechanical behavior.

The total load vector F is split, for the purpose of the control design later, into the vector of external forces FE and the vector of control forces FC:

$$\mathbf{F} = \mathbf{F}_\mathrm{E} + \mathbf{F}_\mathrm{C} = \overline{\mathbf{E}}\overline{\mathbf{f}}(t) + \overline{\mathbf{B}}\overline{\mathbf{u}}(t) = \mathbf{B}_0\mathbf{u} \tag{2}$$

The forces are here generalized quantities, which include also electric charges or electric potentials. Matrices $\overline{\mathbf{E}}$ and $\overline{\mathbf{B}}$ describe the positions of generalized external forces $\overline{\mathbf{f}}$ and the control parameters $\overline{\mathbf{u}}$ in the finite element structure, respectively. Matrix B0 represents the input matrix, and vector u includes all model inputs.

For the controller design purposes equation (2) is accompanied by the output equation in the form:

$$\mathbf{y} = \mathbf{C}_{0q}\mathbf{q} + \mathbf{C}_{0v}\dot{\mathbf{q}} \tag{3}$$

where in a general case $\mathbf{C}_{0q}$ represents the output displacement matrix, and $\mathbf{C}_{0v}$ the output velocity matrix. In the output equation (3) $\mathbf{q}$ represents a generalized displacement vector containing all degrees of freedom defined in the modeling procedure, like in (1). Matrices $\mathbf{C}_{0q}$ and $\mathbf{C}_{0v}$ are obtained through an FE procedure by defining appropriate sensor locations.

Solution of the equation (1) is determined in the form $\mathbf{q}=\boldsymbol{\varphi}^{ej\omega t}$ by solving the eigenvalue problem for a homogeneous case.

The nodal model representation (1) is transformed into a model in modal coordinates applying the following modal transformation:

$$\mathbf{q} = \mathbf{\Phi}\mathbf{q}_m \tag{4}$$

where $\mathbf{q}_m$ represents the vector of modal degrees of freedom or generalized modal displacements and $\mathbf{\Phi}$ is the modal matrix.

Introducing the modal coordinates (4) into (1) after normalization with respect to mass and appropriate transformations, taking into account the orthogonality properties the modal model is obtained, which after introducing the coordinate transformation in the state space form:

$$\mathbf{x} = \begin{bmatrix} \mathbf{\Omega}\mathbf{q}_m \\ \dot{\mathbf{q}}_m \end{bmatrix} \tag{5}$$

can be obtained as a state space realization:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}, \quad \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \tag{6}$$

Considering that flexible structures can be described in terms of independent coordinates, the modal state space model can be expressed in terms of state space realizations ($\mathbf{A}_{mi}$, $\mathbf{B}_{mi}$, $\mathbf{C}_{mi}$) for each mode i (7). With the coordinate transformation as in (5) corresponding matrices in the realization ($\mathbf{A}_{mi}$, $\mathbf{B}_{mi}$, $\mathbf{C}_{mi}$) are determined by [10]:

$$\mathbf{A}_{mi} = \begin{bmatrix} 0 & \omega_i \\ -\omega_i & -2\zeta_i\omega_i \end{bmatrix}, \mathbf{B}_{mi} = \begin{bmatrix} 0 \\ b_{m_i} \end{bmatrix}, \mathbf{C}_{mi} = \begin{bmatrix} \dfrac{c_{mq_i}}{\omega_i} & c_{mv_i} \end{bmatrix} \tag{7}$$

with natural eigenfrequencies ω_i and dampings ζ_i of the eigenmodes. The elements of the realization ($\mathbf{A}_{mi}$, $\mathbf{B}_{mi}$, $\mathbf{C}_{mi}$) are used for assessing the optimal actuator/sensor locations based on candidate input/output transfer functions relating corresponding actuators and sensors.

2.2. Norms – objective functions for optimal placement

Optimization of the actuator/sensor placement in this work is based on the properties of the H_2 and H_∞ norms and approximations for their determining, which enables norm calculation in cases of large structures with high model orders. Exact calculation of the norms in such cases would require high computational effort and computational time. Proposed approach represents a suitable basis for optimal actuator and sensor placement in large structures due to reduced required computational time. The norms and their properties, which are considered and implemented in optimization procedure, are defined for a single mode, for a structure and for a system including a set of actuators and sensors [10], [11]. The main norm properties are summarized below. The proofs are derived in [10].

H_2 norm of a single mode. For a transfer function $G_i(\omega) = \mathbf{C}_{mi}(j\omega\mathbf{I} - \mathbf{A}_{mi})^{-1}\mathbf{B}_{mi}$ of the *i*th mode obtained from the realization (7), the H_2 norm of the mode is estimated as:

$$\|G_i\|_2 \cong \frac{\|\mathbf{B}_{mi}\|_2 \|\mathbf{C}_{mi}\|_2}{2\sqrt{\zeta_i \omega_i}} = \frac{\|\mathbf{B}_{mi}\|_2 \|\mathbf{C}_{mi}\|_2}{\sqrt{2\Delta\omega_i}} \cong \sigma_i \sqrt{2\Delta\omega_i} \tag{8}$$

where $\mathbf{B}_{mi}$, $\mathbf{C}_{mi}$ represent the input and the output matrices of the modal state space model defined in (7), ζ_i is the damping of the i^{th} mode, σ_i the Hankel singular value corresponding to the i^{th} mode, and $\Delta\omega_i = 2\zeta_i\omega_i$ is a frequency segment at the i^{th} resonance for which the value of the power spectrum is one half of its resonance value.

H_∞ norm of a single mode. For an i^{th} mode given by its modal realization ($\mathbf{A}_{mi}$, $\mathbf{B}_{mi}$, $\mathbf{C}_{mi}$) or by the parameters (ω_i, ζ_i, b_{mi}, c_{mi}) the H_∞ norm of the mode is estimated as:

$$\|G_i\|_\infty \cong \frac{\|\mathbf{B}_{mi}\|_2 \|\mathbf{C}_{mi}\|_2}{2\zeta_i \omega_i} = \frac{\|b_{mi}\|_2 \|c_{mi}\|_2}{2\zeta_i \omega_i} \tag{9}$$

H_2 norm of a structure. Given a modal state space realization ($\mathbf{A}_m$, $\mathbf{B}_m$, $\mathbf{C}_m$) of a structure, the H_2 norm of the structure can be determined approximately as the root mean square of the modal norms:

$$\|G\|_2 \cong \sqrt{\sum_{i=1}^{n} \|G_i\|_2^2} \tag{10}$$

where n represents the number of the modes, and G and Gi are the transfer function (or the transfer matrix) of the structure and of the i^{th} mode respectively.

H_∞ norm of a structure. Since the modes are almost independent, the norm H_∞ norm of a structure is approximately determined as the largest of the mode norms:

$$\|G\|_\infty \cong \max_i \|G_i\|_\infty, \quad i = 1, \dots, n \tag{11}$$

For a system including a set of actuators and sensor, for the H_2 and H_∞ norms an additive property both for a single mode and for a structure is valid and can be used in the approximated calculation or the norms.

H_2 and H_∞ norms of a system with a set of actuators and sensors

For a single mode:

$$\|G_i\|_{2,\infty} \cong \sqrt{\sum_{j=1}^{s} \|G_{ij}\|_{(2,\infty)}^2}, \quad i = 1, \dots, n \tag{12}$$

for a structure:

$$\|G\|_{2,\infty} \cong \sqrt{\sum_{j=1}^{s} \|G_j\|_{(2,\infty)}^2} \tag{13}$$

with s representing the number of actuators or the number of sensors, which may be different in a general case.

For a given structure the actuator/sensor placement problem requires the selection of optimal locations as a subset from a given set of possible candidate locations with regard to the specified objective function. The set of possible candidate locations consists of a larger number of elements then the subset of locations to be optimized.

In the first approach the placement is performed based on the placement indices and matrices, where the actuator and sensor placements are solved independently using similar procedures. Definition of placement indices and matrices is based on the additive properties of modal norms on the structural level.

For a flexible structure represented by a modal state space model, the norms of any mode i are determined based on appropriate input ($\mathbf{B}_{mi}$) and output ($\mathbf{C}_{mi}$) matrices of the corresponding mode, (8), (9). If s represents the total number of defined inputs (actuators) $j=1,...,s$, and r the total number of outputs (sensors) $k=1,...,r$, then the corresponding input and output matrices are:

$$\mathbf{B}_{mi}=\left[\mathbf{B}_{mi}^{1} \;\vdots\; \mathbf{B}_{mi}^{2} \;\vdots\; \cdots \;\vdots\; \mathbf{B}_{mi}^{j} \;\vdots\; \cdots \;\vdots\; \mathbf{B}_{mi}^{s}\right],\; \mathbf{C}_{mi}^{\mathrm{T}}=\left[\mathbf{C}_{mi}^{1} \;\vdots\; \mathbf{C}_{mi}^{2} \;\vdots\; \cdots \;\vdots\; \mathbf{C}_{mi}^{k} \;\vdots\; \cdots \;\vdots\; \mathbf{C}_{mi}^{r}\right] \quad (14)$$

where each of the matrices $\mathbf{B}_{mi}^{j}$ represents the 2×1 block of the j^{th} actuator and $\mathbf{C}_{mi}^{k}$ represents the 1×2 block of the k^{th} sensor, both having the form as in (7). Then according to the additive properties of the H_2 and H_∞ norms, the norm of a mode with a set of actuators (sensors) can be approximated by the root mean square sum of the norms of this mode with a single actuator (sensor), which can be expressed as for actuators and sensors respectively as in (15), (16):

$$\left\|G_i\right\|_{(2,\infty)}^{2} \cong \sum_{j=1}^{s}\left\|G_i^{j}\right\|_{(2,\infty)}^{2} \quad (15)$$

$$\left\|G_i\right\|_{(2,\infty)}^{2} \cong \sum_{k=1}^{r}\left\|G_i^{k}\right\|_{(2,\infty)}^{2} \quad (16)$$

Here the H_2 norms of the i^{th} mode with a single actuator corresponding to the j^{th} position, and of the i^{th} mode with a single sensor corresponding to the k^{th} position are given respectively by:

$$\left\|G_i^{j}\right\|_2=\frac{\left\|\mathbf{B}_{mi}^{j}\right\|_2\left\|\mathbf{C}_{mi}\right\|_2}{2\sqrt{\zeta_i\omega_i}},\quad \left\|G_i^{k}\right\|_2=\frac{\left\|\mathbf{B}_{mi}\right\|_2\left\|\mathbf{C}_{mi}^{k}\right\|_2}{2\sqrt{\zeta_i\omega_i}}. \quad (17)$$

Similarly the H_∞ norms of the i^{th} mode with a single actuator corresponding to the j^{th} position, and of the i^{th} mode with a single sensor corresponding to the k^{th} position are expressed as:

$$\left\|G_i^{j}\right\|_\infty=\frac{\left\|\mathbf{B}_{mi}^{j}\right\|_2\left\|\mathbf{C}_{mi}\right\|_2}{2\zeta_i\omega_i},\quad \left\|G_i^{k}\right\|_\infty=\frac{\left\|\mathbf{B}_{mi}\right\|_2\left\|\mathbf{C}_{mi}^{k}\right\|_2}{2\zeta_i\omega_i}. \quad (18)$$

Placement indices are defined in terms of H_2 or H_∞ norms for an actuator or a sensor placement. Each index $\eta_{i(2,\infty)}^{k}$ evaluates the k^{th} actuator (or sensor) in the i^{th} mode in terms of the H_2 or H_∞ norm and it is defined with respect to all modes $i=1,...,n$ and all admissible actuators $k=1,...,s$ (or sensors $k=1,...,r$):

$$\eta^k_{i(2,\infty)} = \frac{\left\| G^k_i \right\|_{(2,\infty)}}{\left\| G \right\|_{(2,\infty)}}. \tag{19}$$

Here the norms $\left\| G^k_i \right\|_{(2,\infty)}$ are determined accordingly as in (17) or (18), and G is the transfer function of the system with all candidate actuators (or sensors). Placement indices determined according to (19) can be arranged in the form of matrix, where each row corresponds to the i^{th} mode and each column to the k^{th} actuator or sensor. Actuator and sensor placement indices are then obtained from the placement matrix by performing column-wise appropriate operations on the elements over all modes. For the objective function in terms of the H_2 norm, actuator (subscript a) or sensor (subscript s) placement indices are determined as the root mean square sum of the column-wise elements:

$$\eta^k_{(a,s)} = sqrt\left(\sum_{i=1}^{n} (\eta^k_i)^2 \right), \quad k = 1,\ldots,p \tag{20}$$

and $p = s$ (for s actuators) or $p = r$ (for r sensors). For the objective function in terms of the H_∞ norm, the actuator/sensor placement index is the largest index over all modes:

$$\eta^k_{(a,s)} = \max_i \left(\eta^k_i \right), \qquad i = 1,\ldots,n, \qquad k = 1,\ldots,p \tag{21}$$

where again $p = s$ (for s actuators) or $p = r$ (for r sensors). The placement indices $\eta^k_{(a,s)}$ determined in this way characterize the importance of the k^{th} actuator or sensor, and represent therefore a criterion for the actuator/sensor placement in the presented approach, which treats the actuator and sensor placement individually.

Placement index for simultaneous actuator/sensor placement is defined as

$$\eta^{jk}_i = \frac{\left\| G^{jk}_i \right\|}{\left\| G^i_m \right\|}, \qquad i = 1,\ldots,n \tag{22}$$

for each mode i, where G^{jk}_i characterizes the i^{th} mode in the presence simultaneously of the actuator placed at the j^{th} candidate location and of the sensor at the k^{th} candidate location.

Besides the introduced placement indices, for the comparison purposes, the controllability index is introduced as an objective function for the optimal placement as well. The influence of the actuators to structural eigenforms is determined by the term $\mathbf{B}_m = \mathbf{\Phi}^{\mathrm{T}}\overline{\mathbf{B}}$, se Eq. (2). Different actuator configurations and their influence on the controllability of the i^{th} mode $\boldsymbol{\varphi}_i$ are investigated by determining the value of $\boldsymbol{\tau}_i(j) = \boldsymbol{\varphi}_i^{\mathrm{T}}\overline{\mathbf{B}}_j$ for the j^{th} actuator location. The controllability index is calculated based on the squared value of $\boldsymbol{\tau}_i$ and divided by the scalar product of the eigenvectors, in order to obtain the controllability index as a measure which is independent of the sign influenced by placement and independent of the eigenvector scaling. The controllability index can thus be determined as [19]:

$$\mu_i(j) = \frac{\boldsymbol{\varphi}_i^{\mathrm{T}}\overline{\mathbf{B}}_j\overline{\mathbf{B}}_j^{\mathrm{T}}\boldsymbol{\varphi}_i}{\boldsymbol{\varphi}_i^{\mathrm{T}}\boldsymbol{\varphi}_i} \tag{23}$$

In a similar way the influence of the sensor placement can be considered through appropriate observability indices for the kth sensor location:

$$\nu_i(k) = \frac{\boldsymbol{\varphi}_i^{\mathrm{T}} \mathbf{C}_k^{\mathrm{T}} \mathbf{C}_k \boldsymbol{\varphi}_i}{\boldsymbol{\varphi}_i^{\mathrm{T}} \boldsymbol{\varphi}_i} \tag{24}$$

3 APPLICATION RESULTS

To illustrate the optimization of the actuator/sensor placement, the results of the placement for a clamped piezoelectric beam and plate are presented in this section.

3.1 Clamped beam

In this example a steel beam clamped on both sides is considered. It is modeled as a 2D beam using the ANSYS software. As a result of the modal analysis, the eigenfrequencies and eigenvectors are determined, which represent an input to the algorithms for the optimal actuator/sensor placement procedures. Meshing the beam along its length results in 101 nodes, and possible candidate positions for this analysis are represented schematically in Figure 1 with pointed nodes 10, 20, … , 90.

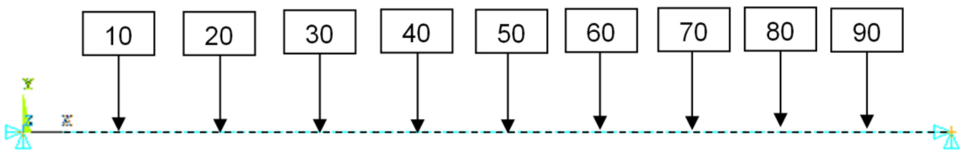

Figure 1. Candidate locations for actuator/sensor placement along the beam clamped on both sides

For the comparison purpose the optimal placement procedure was performed applying the algorithms for separate and simultaneous placement as well as the controllability/ observability indices. Several representative examples are presented below.

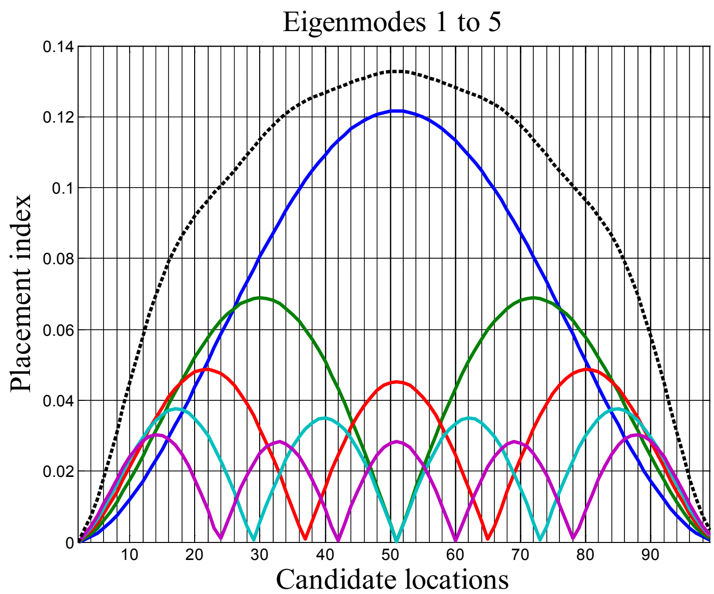

Figure 2. Placement indices calculated based on the H_2 norm for the first five eigenmmodes

Qualitative representations of the curves presenting the values of the placement indices for different positions along the beam are similar for separate placement based on the H_2 and H_∞ norms. Depending on the number of eigenmodes, which should be considered (sensed or actuated) at the same time, the positions for optimal actuator/sensor placement may differ. Figure 2 shows different possible candidate positions with largest placement indices calculated based on the H_2 norm under consideration five bending eigenmodes of interest.

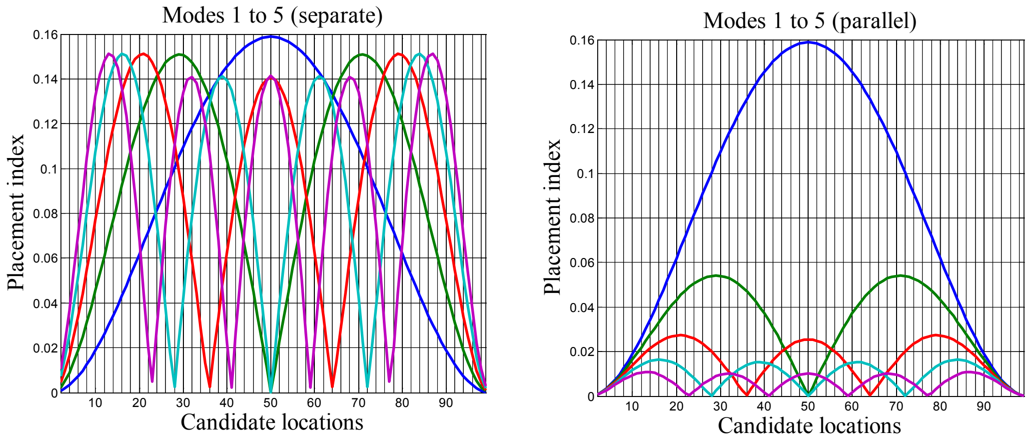

Figure 3. Placement indices based on the H_∞ norm for separate and parallel consideration of the eigenmodes

Placement indices determined based on the H_∞ norm are represented in Figure 3. Left hand side plot in represents the placement indices for individually considered eigenmodes 1 to 5. In the right hand side plot the placement indices were calculated based on parallel consideration of several eigenmodes of interest (here 1 to 5). Locations with largest placement indices indicate the candidates for optimal placement, depending on the number of employed actuators/sensors and on the number of considered modes of interest. Figures 2 and 3 represent the sensor placement indices. The forms of the placement indices curves for actuators are qualitatively the same and for the reason of brevity are omitted here. For the comparison, the method based on the controllability/observability indices is also applied. The results regarding the first five eigenmodes of the beam are summarized by the controllability index representation in Figure 4.

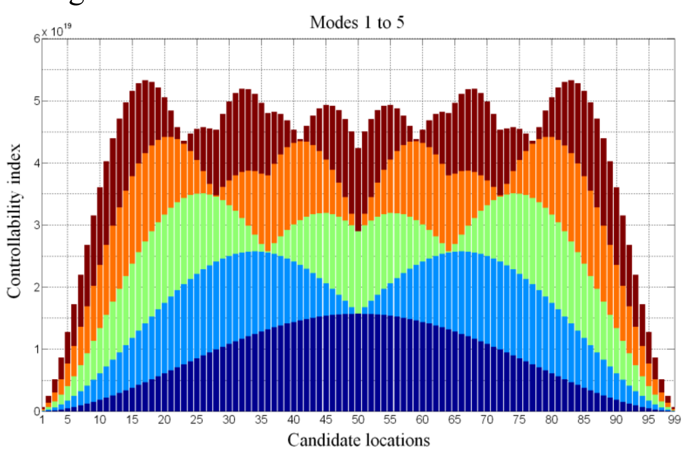

Figure 4. Controllability indices calculated for eigenmodes 1 to 5 for different candidate locations

The results of the three methods applied to the beam clamped on both ends are summarized in Table 1. It can be seen that all three methods provide identical results, when considering eigenmodes individualy. For parallel consideration of several eigenmodes of interest, optimal candidate locations depend on the performance index which was adopted as a criterion for placement.

Table 1. Candidate locations with largest placement indices (beam clamped on both sides)

Modes	Separate placement		Simultaneous placement		Controllability/observability indices
	H_2	H_∞	H_2	H_∞	
1	50	50	50	50	50
2	29, 71	29, 71	29, 71	29, 71	29, 71
3	21, 79	21, 79	21, 79	21, 79	21, 79
4	16, 84	16, 84	16, 84	16, 84	16, 84
5	13, 87	13, 87	13, 87	13, 87	13, 87
1, 2	43 to 57	50			34, 35, 36, 64, 65, 66
1 to 3	48 to 52	50			26, 74
1 to 4	47 to 53	50			21, 79, 41, 59
1 to 5	49 to 51	50			17, 83, 69, 31

3.2 Clamped plate

The plate structure in this example was modeled as a 3D plate in ANSYS software and corresponding eigenvectors of interest were obtained through modal analysis. The meshing of the plate, i.e. the nodes which correspond to candidate locations for actuator/sensor placement are represented in Figure 5. Here the corresponding rows and columns are numerated for a better preview.

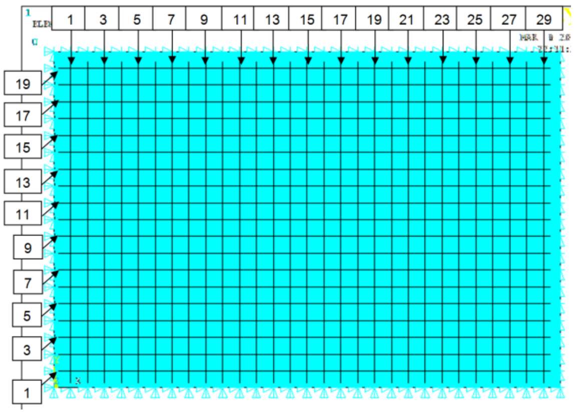

Figure 5. Candidate locations for the plate denoted by corresponding row and column numbers

Due to a very high number of nodes, i.e. candidate locations for the plate, the simultaneous placement procedure would not give a clear representation and therefore it is omitted from this analysis. The results of other two methods, separate placement and controllability index, are compared and summarized in Table 2. Besides, several representative results of the actuator/sensor placement for the clamped plate are shown in the figures below. Complete agreement of the results is available for individual consideration of the eigenmodes. For parallel consideration of several structural eigenmodes of interest, the arising differences are based on the calculation, i.e. on the definition of the placement indices for the structure. Qualitative representations of the placement indices based on H_2 and H_∞ norms as well as of the controllability index for individually considered modes are the same. Actuator placement indices based on the H_2 norm for selected individual modes are represented in Figure 6.

Table 2. Candidate locations with largest placement indices (plate)

Modes	Separate placement		Controllability index
	H_2	H_∞	
1	(10,15)	(10,15)	(10,15)
2	(10,8), (10,9), (10,21), (10,22)	(10,8), (10,9), (10,21), (10,22)	(10,8), (10,9), (10,21), (10,22)
3	(14,15), (6,15)	(14,15), (6,15)	(14,15), (6,15)
4	(10,6), (10,15), (10,24)	(10,6), (10,15), (10,24)	(10,6), (10,15), (10,24)
5	(6,8), (6,22), (14,8), (14,22)	(6,8), (6,22), (14,8), (14,22)	(6,8), (6,22), (14,8), (14,22)
1, 2	(10,9), (10,10), (10,20), (10,21)	(10,15)	(10,10), (10,20)
1 to 3	(9,9), (9,10), (9,20), (9,21)	(10,15)	(7,11), (7,19), (13,11), (13,19)
1 to 4	(10,7), (10,22)	(10,15)	(12,8), (8,8), (12,22), (8,22)
1 to 5	(7,8), (13,8), (7,22), (13,22)	(10,15)	(13,8), (7,8), (13,22), (7,22)

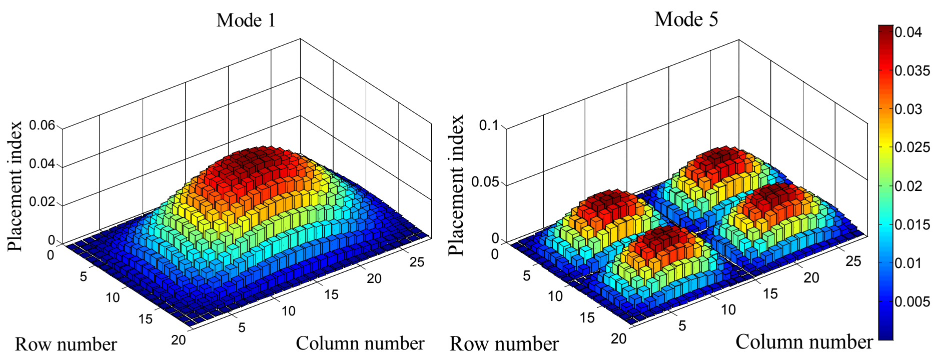

Figure 6. Placement indices based on H_2 norm for individually considered selected eigenmodes of the plate

Figure 7 represents the values of the placement indices calculated for all selected candidate locations based on the H_2 norm under parallel consideration of several eigenmodes of interest (*left*: modes 1 and 2; *right:* modes 1 to 5).

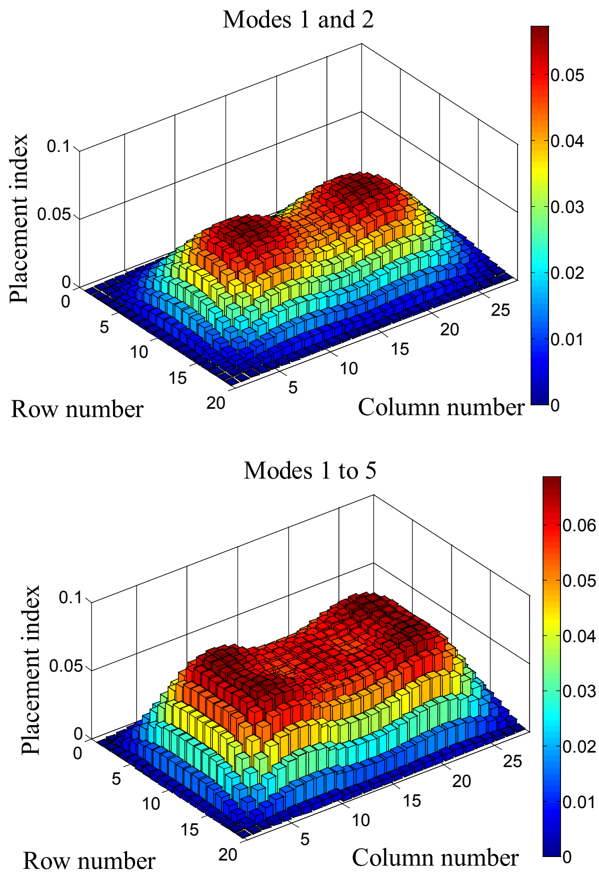

Figure 7. Placement indices for the plate based on the H_2 norm (parallel consideration of eigenmodes of interest)

4 CONCLUSIONS

In this paper the optimization methods for actuator/sensor placement for large flexible structures are presented, based on balanced reduction of structural models. Balanced modal reduction of the model orders for structures with large numbers of degrees of freedom is proposed as an efficient modeling procedure, which results in a realization with equally controllable and observable retained states. Optimal placement procedure is based on the properties of the H_2 and H_∞ norms and approximations for their determining. Proposed approach represents a suitable basis for optimal actuator and sensor placement in large structures due to reduced required computational time.

Optimization procedure is proven by showing examples of a beam clamped on both sides and clamped plate. For these examples an extensive analysis was conducted and systematized results of separate and simultaneous placement procedures for individual and parallel consideration of the structural modes are shown. The efficiency of the proposed method is also proven by the comparison with the optimization results based on controllability and observability indices. This analysis has shown a compete agreement of the results. The method suggested in this paper also covers a broad spectrum of possible problems, which do not have to be necessarily limited only to piezoelectric actuators and sensors, but can be extended more generally to systems with integrated actuators and sensors, whose effect may be considered through actuation forces or moments.

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